FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:
- **(i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
- **(ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- **(iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- **(iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- **(v)** Extra attempt of any question or any part of the attempted question will not be considered.
- **(vi)** **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let H, K be subgroups of a group G. Prove that HK is a subgroup of G if and only if $HK=KH$. (10)

(b) If N, M are normal subgroups of a group G, prove that (10) **(20)**

$$NM/M \cong N/N\cap M.$$

Q. 2. **(a)** If R is a commutative ring with unit element and M is an ideal of R then show that M is a maximal ideal of R if and only if R/M is a field. (10)

(b) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then show that we can find elements a and b in F such that (10) **(20)**

$$1 + \alpha a^2 + \beta b^2 = 0.$$

Q. 3. **(a)** Let V be a finite-dimensional vector space over a field F and W be a subspace of V. Then show that W is finite-dimensional, (10)

$$\dim W \leq \dim V \text{ and } \dim V/W = \dim V - \dim W.$$

(b) Suppose V is a finite-dimensional vector space over a field F. Prove that a linear transformation $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. (10) **(20)**

SECTION-B

Q. 4. **(a)** Use the Mean-Value Theorem to show that if f is differentiable on an interval I, and if $|f'(x)| \leq M$ for all values of x in I, then (10)

$$|f(x) - f(y)| \leq M|x - y|$$

for all values of x and y in I. Use this result to show further that

$$|\sin x - \sin y| \leq |x - y|.$$

(b) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable at t, and if $z = f(x, y)$ is differentiable at the point $(x, y) = \big(x(t), y(t)\big)$, then $z = f\big(x(t), y(t)\big)$ is differentiable at t and (10) **(20)**

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y).

$\iint \quad x - \quad) \quad dy$

Q. 6. **(a)** Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)

(b) Show that if a, b and c are nonzero, then the plane whose intercepts with the coordinate axes are $x = a$, $y = b$, and $z = c$ is given by the equation. (10) **(20)**

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1.$$

SECTION-C

Q. 7. **(a)** Prove that a necessary and sufficient condition that (10)

$$w = f(z) = u(x, y) + iv(x, y)$$

be analytic in a region R is that the Cauchy-Riemann equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \text{and} \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

are satisfied in R where it is supposed that these partial derivatives are continuous in R.

(b) Show that the function $f(z) = \bar{z}$ is not analytic anywhere in the complex plane Z. (10) **(20)**

Q. 8. **(a)** Let $f(z)$ be analytic inside and on the boundary C of a simply-connected region R. Prove that (10)

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2} dz.$$

(b) Show that (10) **(20)**

$$\int_0^{2\pi} \frac{d\theta}{(5-3\sin\theta)^2} = \frac{5\pi}{32}.$$

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